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Reflection and transmission of a transient, elastic, line-source excited SH wave by a planar, elastic bonding surface in a solid

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Dedicated to Professor Jan D. Achenbach on the occasion of his being the recipient of the William Prager Medal of the Society of Engineering Science

Abstract

Closed-form time-domain expressions are obtained for the particle displacement of the elastic wave motion generated by a two-dimensional SH-wave line source and reflected and transmitted by a planar, elastic bonding interface of two homogeneous, isotropic, semi-infinite, perfectly elastic solids. The properties of the elastic bonding interface are characterized by a matrix of 'spring coefficients' through which the traction on each of the two faces is linearly related to the particle displacement of either of the two faces. The solution is constructed with the aid of (an extension of) the modified Cagniard method. The obtained solution of the forward model is believed to be of importance to the inverse problem that aims at reconstructing the elements of the matrix of 'spring coefficients' from measured values of the reflected and/or the transmitted wavefield quantities at a number of positions.

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Keywords: Elastic-wave reflection/transmission; Non-destructive testing; Mechanical bonding

1. Introduction

In this paper the elastic-wave reflection and transmission properties of an elastic interfacial bonding of two semi-infinite solids are investigated for the simplest possible case of a line-source excited two-dimensional SH-wave. The interfacial bonding is considered to be of vanishing thickness, which implies that the travel time of elastic waves to traverse it is negligible with respect to the pulse time width of the exciting wave motion. As a consequence, in the direction normal to the plane of the bonding the interface acts as a

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linear, time-invariant, passive, instantaneously reacting, elastostatic system. The elastic properties of such a system are expressed by a local linear relationship between the tractions at either side of the interface on the one hand and the particle displacements at either side of the interface on the other hand. The coefficients entering into this relationship form a matrix of 'spring coefficients'. From an elastodynamic point of view such a boundary condition can be considered as a generalization of the linear-slip boundary condition that models an imperfect fracture. The transient elastic wave reflection and transmission properties of such a fracture have recently been investigated by Verweij and Chapman (1999). In their paper, references to other literature on the use of linear-slip boundary conditions to describe the properties of fractures can be found. The present boundary condition has also served to investigate the diffraction of a plane SH-wave by a generalized linear-slip fracture of bounded extent in the Kirchhoff approximation (De Hoop, 2000). Physical constraints on the 'spring coefficients' are that their values must satisfy the principle of reciprocity and that their matrix must be positive definite.

The author's modification of Cagniard's method to solve transient wave propagation and diffraction problems (De Hoop, 1958, 1960, 1988a,b; Achenbach, 1973; Miklowitz, 1978; Aki and Richards, 1980) (or actually an extension of it that has also been employed by Verweij and Chapman (1999)) is used to obtain analytic, closed-form expressions for the particle displacement of the generated SH-wave motion.

The characterization of an elastic interfacial bonding by a matrix of spring coefficient opens the possibility of applying inverse-scattering optimization methods to reconstruct the properties of the bonding from observed reflected and transmitted wave data, expressed in terms of these coefficients. (For some general aspects of procedures of this kind, see De Hoop and De Hoop, 2000.) The results of the present study are therefore believed to be of importance in the quantitative non-destructive monitoring of interfacial bondings in mechanical structures.

2. Description of the configuration

The interfacial bonding under consideration joins two semi-infinite homogeneous, isotropic, perfectly elastic solids with volume density of mass ρ and Lamé stiffness coefficients λ and μ. The corresponding SH-wave speed is $c_S = (\mu/\rho)^{1/2}$. Position in the configuration is specified by the coordinates $\{x, y, z\}$ with respect to an orthogonal, Cartesian reference frame with the origin $\mathcal{O}$ and the three mutually perpendicular base vectors $\{\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z\}$ of unit length each. In the indicated order, the base vectors form a right-handed system. The time coordinate is t. Partial differentiation is denoted by ∂. The interfacial bonding coincides with the plane $\{z = 0\}$. The configuration is shown in Fig. 1. The line source generating the SH-wave motion is located at $\{x = 0, z = h\}$ with $h > 0$. The particle displacement of the generated two-dimensional wave motion is parallel to the y-axis.

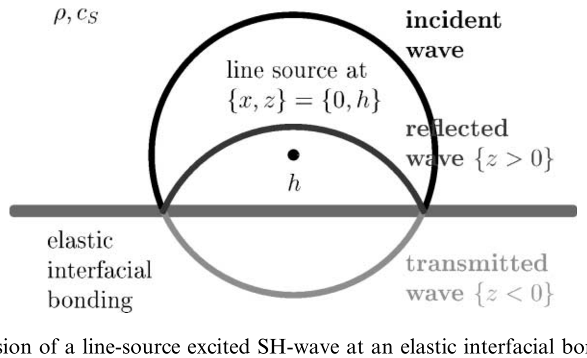

Fig. 1. Reflection and transmission of a line-source excited SH-wave at an elastic interfacial bonding of two semi-infinite solids.

3. Formulation of the problem

Let the area density of the exciting force be given by

$$\boldsymbol{f} = F(t)\delta(x, z-h)\boldsymbol{i}_y, \tag{1}$$

where $F(t)$ has the temporal support $\{t \in \mathbb{R}, t \geqslant 0\}$ and $\delta(x, z-h)$ is the two-dimensional Dirac distribution operative at $\{x = 0, z = h\}$. The particle displacement $u_y = u_y(x, z, t)$ of the generated SH-wave motion then satisfies the two-dimensional wave equation

$$(\partial_x^2 + \partial_z^2 - c_S^{-2}\partial_t^2)u_y = -\mu^{-1}\partial_t F(t)\delta(x, z-h) \quad \text{for} \quad z \neq 0, \tag{2}$$

together with the initial conditions $u_y(x, z, t) = 0$ and $\partial_t u_y(x, z, t) = 0$ for $t < 0$. The total wave motion is decomposed into the *incident wave* $u_y^{\mathrm{i}} = u_y^{\mathrm{i}}(x, z, t)$ with the spatial support $\mathbb{R}^2$, the *reflected wave* $u_y^{\mathrm{r}} = u_y^{\mathrm{r}}(x, z, t)$ with the spatial support $\{z > 0\}$ and the *transmitted wave* $u_y^{\mathrm{t}} = u_y^{\mathrm{t}}(x, z, t)$ with the spatial support $\{z < 0\}$, according to

$$u_y = u_y^{\mathrm{i}} + u_y^{\mathrm{r}} \quad \text{for} \quad z > 0, \tag{3}$$

$$u_y = u_y^{\mathrm{t}} \quad \text{for} \quad z < 0. \tag{4}$$

The tractions $\tau_{z,y}^{\mathrm{i,r,t}} = \tau_{z,y}^{\mathrm{i,r,t}}(x, z, t)$ normal to planes parallel to the plane of the interface associated with the different wave constituents are related to their particle displacement counterparts via the stress/strain relation

$$\tau_{z,y}^{\mathrm{i,r,t}} = \mu\partial_z u_y^{\mathrm{i,r,t}} \quad \text{for } z \neq 0. \tag{5}$$

The boundary condition used to characterize the elastodynamic properties of the elastic interfacial bonding are taken as

$$\begin{bmatrix} \tau_{z,y}(x, 0+, t) \\ -\tau_{z,y}(x, 0-, t) \end{bmatrix} = \begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{bmatrix} \begin{bmatrix} u_y(x, 0+, t) \\ u_y(x, 0-, t) \end{bmatrix}, \tag{6}$$

where $0-$ is a shorthand notation for $\lim_{z\uparrow 0}$ and $0+$ is a shorthand notation for $\lim_{z\downarrow 0}$. The coefficients in this relation can be considered as a kind of *spring coefficients*, which are quantitative representatives of the properties of the bonding considered as a linear, time-invariant, passive, instantaneously reacting elastostatic system. In view of the principle of reciprocity we have

$$C_{1,2} = C_{2,1}, \tag{7}$$

while the property of passivity leads to the conditions

$$C_{1,1} > 0, \quad C_{2,2} > 0, \quad \begin{vmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{vmatrix} > 0. \tag{8}$$

The values $C_{1,1} = C_{1,2} = C_{2,1} = C_{2,2} = 0$ yield the case where, on both faces of the interface, the traction vanishes, while the values $C_{1,1} = -C_{1,2} = -C_{2,1} = C_{2,2}$ yield the case considered by Verweij and Chapman (1999) of the interface characterized by a linear-slip boundary condition.

4. Determination of the complex slowness-plane wave amplitudes

The problem will be solved with the aid of (an extension of) the modified Cagniard method. In this method, first the Laplace transformation with respect to time is carried out. To show the notation, we give the expression for the particle displacement

$$\hat{u}_y(x,z,s) = \int_{t=0}^{\infty} \exp(-st) u_y(x,z,t)\,\mathrm{d}t. \tag{9}$$

For the physically interesting case of bounded time signatures of the exciting force, the right-hand side of Eq. (9) exists in the right half $\{s \in \mathbb{C}; \mathrm{Re}(s) > 0\}$ of the complex s-plane, where it is a regular, analytic function of the *complex frequency* s. Lerch's theorem of the one-sided Laplace transformation (Widder, 1946) states that a causal time function belonging to a class of functions that encompasses the one we have specified, is in a one-to-one manner related to its Laplace transform at the equidistant set of points $\{s \in \mathbb{R}; s = s_0 + nh, s_0 > 0, h > 0, n = 0,1,2,\ldots\}$ on the positive real s-axis. With this property in mind, the Laplace transform parameter s will, once and for all in our analysis, be restricted to real, positive values.

Next, to account for the interfacial boundary conditions, we introduce the complex-slowness representations

$$\hat{u}^{\mathrm{i}}_y(x,z,s) = \frac{s\hat{F}(s)}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{1}{2\mu\gamma_{\mathrm{S}}(p)} \exp\{-s[px + \gamma_{\mathrm{S}}(p)|z-h|]\}\,\mathrm{d}p \quad \text{for all } z, \tag{10}$$

which follows from Eq. (2), together with

$$\hat{u}^{\mathrm{r}}_y(x,z,s) = \frac{s\hat{F}(s)}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{R(p,s)}{2\mu\gamma_{\mathrm{S}}(p)} \exp\{-s[px + \gamma_{\mathrm{S}}(p)(h+z)]\}\,\mathrm{d}p \quad \text{for } z > 0. \tag{11}$$

$$\hat{u}^{\mathrm{t}}_y(x,z,s) = \frac{s\hat{F}(s)}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{T(p,s)}{2\mu\gamma_{\mathrm{S}}(p)} \exp\{-s[px + \gamma_{\mathrm{S}}(p)(h-z)]\}\,\mathrm{d}p \quad \text{for } z < 0. \tag{12}$$

Here, i is the imaginary unit and

$$\gamma_{\mathrm{S}} = (c_{\mathrm{S}}^{-2} - p^2)^{1/2} \quad \text{with} \quad \mathrm{Re}(\gamma_{\mathrm{S}}) \geqslant 0 \quad \text{for all } p \in \mathbb{C}. \tag{13}$$

The corresponding representations for the traction normal to the planes parallel to the plane of the interface follow from Eqs. (5), (10)–(12) as

$$\hat{\tau}^{\mathrm{i}}_{z,y}(x,z,s) = -\frac{s^2\hat{F}(s)}{4\pi\mathrm{i}} \mathrm{sign}(z-h) \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \exp\{-s[px + \gamma_{\mathrm{S}}(p)|z-h|]\}\,\mathrm{d}p \quad \text{for all } z, \tag{14}$$

$$\hat{\tau}^{\mathrm{r}}_{z,y}(x,z,s) = -\frac{s^2\hat{F}(s)}{4\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} R(p,s) \exp\{-s[px + \gamma_{\mathrm{S}}(p)(z+h)]\}\,\mathrm{d}p \quad \text{for } z > 0, \tag{15}$$

$$\hat{\tau}^{\mathrm{t}}_{z,y}(x,z,s) = \frac{s^2\hat{F}(s)}{4\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} T(p,s) \exp\{-s[px + \gamma_{\mathrm{S}}(p)(h-z)]\}\,\mathrm{d}p \quad \text{for } z < 0. \tag{16}$$

The complex slowness-plane reflection coefficient $R = R(p,s)$ and the complex slowness-plane transmission coefficient $T = T(p,s)$ remain to be determined from the boundary conditions in the plane $\{z = 0\}$ of the bonding. Substitution of the representations (10)–(16) into Eq. (6) leads to

$$s\mu\gamma_{\mathrm{S}}(p)[1 - R(p,s)] = C_{1,1}[1 + R(p,s)] + C_{1,2}T(p,s), \tag{17}$$

$$-s\mu\gamma_{\mathrm{S}}(p)T(p,s) = C_{2,1}[1 + R(p,s)] + C_{2,2}T(p,s), \tag{18}$$

from which we obtain

$$R(p,s) = 1 - \frac{2[\mu s\gamma_{\mathrm{S}}(p)C_{1,1} + \Delta_{\mathrm{C}}]}{\Delta(p,s)}, \tag{19}$$

$$T(p,s) = -\frac{2\mu s \gamma_S(p) C_{2,1}}{\Delta(p,s)}, \tag{20}$$

with

$$\Delta(p,s) = [\mu s \gamma_S(p) + C_{1,1}][\mu s \gamma_S(p) + C_{2,2}] - C_{1,2}C_{2,1} \tag{21}$$

and

$$\Delta_C = C_{1,1}C_{2,2} - C_{1,2}C_{2,1}. \tag{22}$$

For $\gamma_S(p) = 0$, i.e. for $p = \pm c_S^{-1}$, we obtain $R(p,s) = -1$ and $T(p_0,s) = 0$. For $\gamma_S(p) \neq 0$, i.e. for $p \neq \pm c_S^{-1}$, we rewrite the right-hand sides of Eqs. (19) and (20) as their partial-fraction decompositions. To this end, Eq. (21) is rewritten as

$$\Delta(p,s) = [\mu \gamma_S(p)]^2 [s + \alpha_1(p)][s + \alpha_2(p)], \tag{23}$$

with

$$\alpha_1(p) = \frac{1}{\mu \gamma_S(p)} \left\{ \frac{C_{1,1} + C_{2,2}}{2} - \left[\left(\frac{C_{1,1} + C_{2,2}}{2} \right)^2 - \Delta_C \right]^{1/2} \right\}, \tag{24}$$

$$\alpha_2(p) = \frac{1}{\mu \gamma_S(p)} \left\{ \frac{C_{1,1} + C_{2,2}}{2} + \left[\left(\frac{C_{1,1} + C_{2,2}}{2} \right)^2 - \Delta_C \right]^{1/2} \right\}. \tag{25}$$

It is easily verified that, in view of the conditions laid upon the interfacial spring coefficients, the expressions in braces in both α_1 and α_2 are real-valued. For the reflection coefficient the partial-fraction decomposition leads to

$$R(p,s) = 1 - \frac{R_1(p)}{s + \alpha_1(p)} - \frac{R_2(p)}{s + \alpha_2(p)}, \tag{26}$$

with

$$R_1(p) = \frac{2[-\mu \gamma_S(p)\alpha_1(p) C_{1,1} + \Delta_C]}{[\mu \gamma_S(p)]^2 [\alpha_2(p) - \alpha_1(p)]} \tag{27}$$

and

$$R_2(p) = \frac{2[\mu \gamma_S(p)\alpha_2(p) C_{1,1} - \Delta_C]}{[\mu \gamma_S(p)]^2 [\alpha_2(p) - \alpha_1(p)]}, \tag{28}$$

and for the transmission coefficient to

$$T(p,s) = \frac{T_1(p)}{s + \alpha_1(p)} - \frac{T_2(p)}{s + \alpha_2(p)}, \tag{29}$$

with

$$T_1(p) = \frac{2\alpha_1(p) C_{2,1}}{\mu \gamma_S(p)[\alpha_2(p) - \alpha_1(p)]} \tag{30}$$

and

$$T_2(p) = \frac{2\alpha_2(p) C_{2,1}}{\mu \gamma_S(p)[\alpha_2(p) - \alpha_1(p)]}. \tag{31}$$

These results will be used to construct the time-domain expressions for the particle displacements of the reflected and transmitted waves.

5. Transformation back to the time domain

In this section, the Laplace transformed reflected and transmitted wave particle displacements will be transformed back to the time domain through the application of (an extension of) the modified Cagniard method. The starting points are the representations (10)–(12), in which for the complex slowness domain reflected- and transmitted-wave amplitudes $R(p,s)$ and $T(p,s)$ the expressions (26) and (29) are substituted.

The first step consists of replacing the original path of integration in the complex p-plane (the imaginary axis) by a path along which the exponential functions in the integrands take the form $\exp(-s\tau)$, where τ is a real variable of integration. Once this has been accomplished (for details, see Appendix A), the application of Lerch's theorem in conjunction with some standard rules of the one-sided Laplace transformation suffice to construct the final time-domain expressions for the particle displacement. Elements in this procedure are the convolution theorem and the inverse Laplace Transforms

$$\texttt{InverseLaplaceTransform}[\exp(-s\tau)] = \delta(t-\tau) \tag{32}$$

$$\texttt{InverseLaplaceTransform}\left[\frac{\exp(-s\tau)}{s+\beta(\tau)}\right] = \exp[-\beta(\tau)(t-\tau)]H(t-\tau), \tag{33}$$

where $H(t)$ is the Heaviside unit step function. The different wave constituents are discussed separately below.

5.1. Incident wave

Using Eq. (A.9) in Eq. (10), the time-domain expressions for the incident wave is obtained as

$$u^{\mathrm{i}}(x,z,t) = \frac{\partial_t F(t)}{\mu} \overset{(t)}{*} G^{\mathrm{i}}(x,z,t) \quad \text{for all } z, \tag{34}$$

in which $\overset{(t)}{*}$ denotes time convolution and

$$G^{\mathrm{i}}(x,z,t) = \frac{1}{2\pi\left(t^2 - T_{\mathrm{S}}^{\mathrm{i}^2}\right)^{1/2}} H(t - T_{\mathrm{S}}^{\mathrm{i}}), \tag{35}$$

with

$$T_{\mathrm{S}}^{\mathrm{i}} = [x^2 + (z-h)^2]^{1/2} \tag{36}$$

as the SH-wave travel time from the location of the source to the point of observation, is the well-known Green's function of the two-dimensional scalar wave equation.

5.2. Reflected wave

Using Eq. (A.9) in Eqs. (11) and (26), the time-domain expression for the reflected wave is obtained as

$$u^{\mathrm{r}}(x,z,t) = \frac{\partial_t F(t)}{\mu} \overset{(t)}{*} G^{\mathrm{r}}(x,z,t) \quad \text{for } z > 0, \tag{37}$$

in which

$$G^{\mathrm{r}}(x,z,t) = \left[\int_{\tau=T_{\mathrm{S}}^{\mathrm{r}}}^{t} \mathrm{Re}\{\delta(t-\tau) - R_1(p_{\mathrm{S}}^{\mathrm{r}}) \exp[-\alpha_1(p_{\mathrm{S}}^{\mathrm{r}})(t-\tau)] - R_2(p_{\mathrm{S}}^{\mathrm{r}}) \times \exp[-\alpha_2(p_{\mathrm{S}}^{\mathrm{r}})(t-\tau)]\} \frac{1}{2\pi(\tau^2 - T_{\mathrm{S}}^{\mathrm{r}2})^{1/2}} \mathrm{d}\tau \right] H(t - T_{\mathrm{S}}^{\mathrm{r}}), \tag{38}$$

with

$$p_{\mathrm{S}}^{\mathrm{r}} = \frac{x\tau}{r^{\mathrm{r}2}} + \mathrm{i}\frac{z+h}{r^{\mathrm{r}2}}\left(\tau^2 - \frac{r^{\mathrm{r}2}}{c_{\mathrm{S}}^2}\right)^{1/2}, \tag{39}$$

$$r^{\mathrm{r}} = [x^2 + (z+h)^2]^{1/2}, \tag{40}$$

and

$$T_{\mathrm{S}}^{\mathrm{r}} = r^{\mathrm{r}}/c_{\mathrm{S}} \tag{41}$$

as the SH-wave travel time from the image of the location of the source in the interface to the point of observation in the half-space $\{z > 0\}$.

5.3. Transmitted wave

Using Eq. (A.9) in Eqs. (12) and (29), the time-domain expression for the transmitted wave is obtained as

$$u^{\mathrm{t}}(x,z,t) = \frac{\partial_t F(t)}{\mu} \overset{(t)}{*} G^{\mathrm{t}}(x,z,t) \quad \text{for } z < 0, \tag{42}$$

in which

$$G^{\mathrm{t}}(x,z,t) = \left[\int_{\tau=T_{\mathrm{S}}^{\mathrm{t}}}^{t} \mathrm{Re}\{T_1(p_{\mathrm{S}}^{\mathrm{t}}) \exp[-\alpha_1(p_{\mathrm{S}}^{\mathrm{t}})(t-\tau)] - T_2(p_{\mathrm{S}}^{\mathrm{t}}) \exp[-\alpha_2(p_{\mathrm{S}}^{\mathrm{t}})(t-\tau)]\} \frac{1}{2\pi(\tau^2 - T_{\mathrm{S}}^{\mathrm{t}2})^{1/2}} \mathrm{d}\tau \right] \times H(t - T_{\mathrm{S}}^{\mathrm{t}}), \tag{43}$$

with

$$p_{\mathrm{S}}^{\mathrm{t}} = \frac{x\tau}{r^{\mathrm{t}2}} + \mathrm{i}\frac{h+|z|}{r^{\mathrm{t}2}}\left(\tau^2 - \frac{r^{\mathrm{t}2}}{c_{\mathrm{S}}^2}\right)^{1/2}, \tag{44}$$

$$r^{\mathrm{t}} = \left[x^2 + (h+|z|)^2\right]^{1/2}, \tag{45}$$

and

$$T_{\mathrm{S}}^{\mathrm{t}} = r^{\mathrm{t}}/c_{\mathrm{S}} \tag{46}$$

as the SH-wave travel time from the location of the source in the half-space $\{z > 0\}$ to the point of observation in the half-space $\{z < 0\}$.

6. Discussion of the results

The expressions for the incident, reflected and transmitted particle displacement constituents of the generated SH-wave motion obtained in Section 5 are of the closed-form, analytic type. In them, the convolution integral with the source signature $F = F(t)$ and the integrations with respect to τ in the expressions for the Green's functions G^r and G^t need be evaluated numerically. The physical behavior of the interfacial bonding manisfests itself via the expressions for α_1, α_2, R_1, R_2, T_1 and T_2 obtained in Section 4. To illustrate the kind of transient behavior that can be expected, the case of a symmetrically behaving bonding, for which $C_{1,1} = C_{2,2}$, is further investigated. Let for this case $C_{1,1} = C_{2,2} = C_1 > 0$ and let further $C_{1,2} = C_{2,1} = C_2$, with, in view of the condition of passivity, $|C_2| < C_1$. Then,

$$\alpha_1 = \frac{C_1 - |C_2|}{\mu\gamma_S(p)}, \tag{47}$$

$$\alpha_2 = \frac{C_1 + |C_2|}{\mu\gamma_S(p)}, \tag{48}$$

$$R_1 = \frac{C_1 - |C_2|}{\mu\gamma_S(p)}, \tag{49}$$

$$R_2 = \frac{C_1 + |C_2|}{\mu\gamma_S(p)}, \tag{50}$$

$$T_1 = \mathrm{sign}(C_2)\frac{C_1 - |C_2|}{\mu\gamma_S(p)}, \tag{51}$$

$$T_2 = \mathrm{sign}(C_2)\frac{C_1 + |C_2|}{\mu\gamma_S(p)}. \tag{52}$$

In the following some numerical results are presented for an exciting force having the unipolar time signature

$$F(t) = F_0(t/t_r)\exp(-t/t_r + 1)H(t), \tag{53}$$

where F_0 is the *amplitude* (maximum value) of the force, t_r is the *pulse rise time* (defined through $F(t_r) = F_0$) and $H(t)$ is the Heaviside unit step function. The *pulse time width* associated with (53) (defined through $\int_{t=0}^{\infty} F(t)\,dt = F_0 t_w$) is $t_w = t_r \exp(1)$. Fig. 2 shows this force source signature for $F_0 = 1$ N and $t_r = 10$ μs. The line source is located at $x = 0$, $z = h = 10$ mm and is present in a medium with shear modulus $\mu = 2 \times 10^{11}$ Pa and S wave speed $c_S = 2 \times 10^3$ m/s. The 'spring coefficients' characterizing the bonding have been chosen as $C_1 = -C_2 = 2 \times 10^{11}$ N/m. These values have some relationship to a much stiffer bonding material than the adjacent material and a thickness in the order of 1 mm under the quasi-static assumption of a linearly

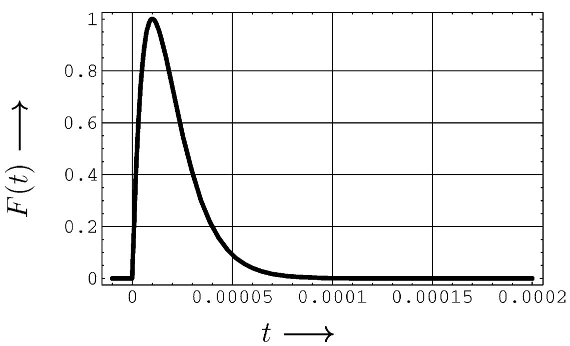

Fig. 2. Exciting force source signature ($F_0 = 1$ N, $t_r = 10$ μs).

varying shear stress across the bonding. The incident and reflected waves are computed at $x = 0$, $z = 0.2$ m. Fig. 3 shows the incident-wave Green's function, Fig. 4 gives the reflected-wave Green's function. Figs. 5 and 6 show the corresponding total wave fields. The transmitted wave is computed at $x = 0$, $z = -0.10$ m. Fig. 7 shows the transmitted-wave Green's function, Fig. 8 the corresponding total wavefield.

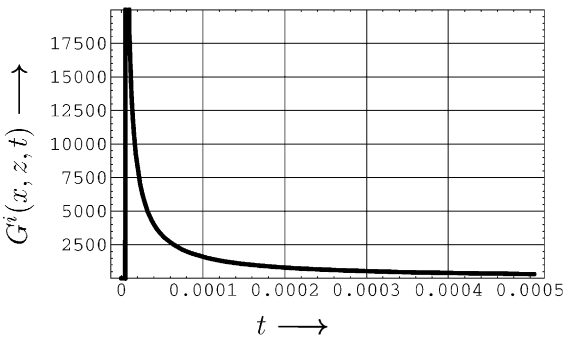

Fig. 3. Incident-wave Green's function at $x = 0$ m, $z = 20$ mm.

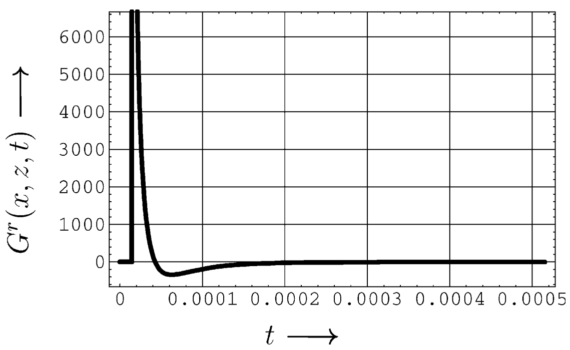

Fig. 4. Reflected-wave Green's function at $x = 0$ m, $z = 20$ mm.

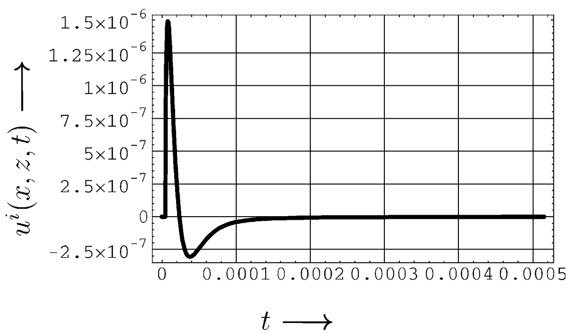

Fig. 5. Incident wave at $x = 0$ m, $z = 20$ mm.

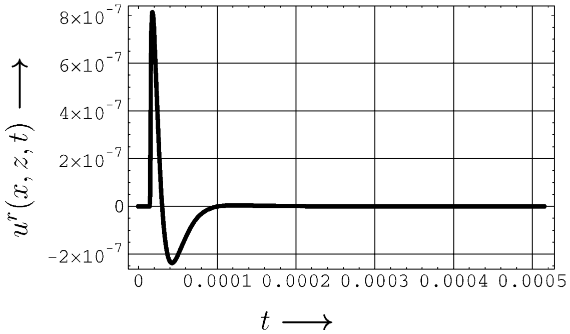

Fig. 6. Reflected wave at $x = 0$ m, $z = 20$ mm.

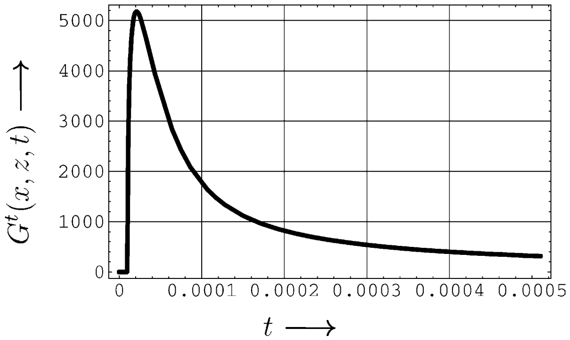

Fig. 7. Transmitted-wave Green's function at $x = 0$ m, $z = -10$ mm.

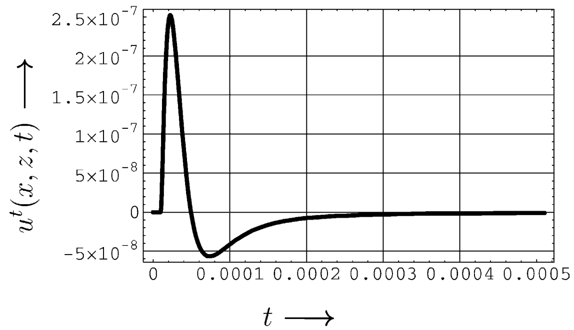

Fig. 8. Transmitted wave at $x = 0$ m, $z = -10$ mm.

7. Conclusion

Closed-form time-domain expressions have been obtained for the particle displacement of the elastic wave motion generated by a two-dimensional SH-wave line source and reflected and transmitted by a planar, elastic bonding interface of two homogeneous, isotropic, semi-infinite, perfectly elastic solids. The properties of the elastic bonding interface are characterized by a matrix of 'spring coefficients' through which the traction on each of the two faces is linearly related to the particle displacement of either of the two faces. The solution has been constructed with the aid of (an extension of) the modified Cagniard method. The obtained solution of the forward model is believed to be of importance to the inverse problem that aims at reconstructing the elements of the matrix of 'spring coefficients' from measured values of the reflected and/or the transmitted wavefield quantities at a number of positions. To generate the numerical results certain plausable values of the 'spring coefficients' of the bonding have been used. What these values turn out to be for bonding surfaces met in practice needs further investigation.

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Appendix A. The modified Cagniard path and its properties

In this appendix the steps related to replacing the integration along the imaginary axis in the complex slowness plane by one along the modified Cagniard path will be briefly reviewed. The generic form of the relevant complex slowness representation is taken as

$$\hat{w}(x,z,s) = \frac{1}{2\pi i}\int_{p=-i\infty}^{i\infty} \frac{A(p,s)}{2\gamma_S(p)} \exp\{-s[px+\gamma_S(p)H]\}\,dp, \tag{A.1}$$

where $A = A(p,s)$ is some amplitude function of p and s that is analytic in the entire complex p-plane cut along the branch cuts associated with $\gamma_S = \gamma_S(p)$, with branch points at $p = \pm c_S^{-1}$, and of order $O(1)$ or lower as $|p| \to \infty$. Note that the expressions (26) for $R(p,s)$ and (29) for $T(p,s)$ satisfy these conditions. Furthermore, $H > 0$ is some propagation path in the direction normal to the interface.

As stipulated in Section 5, the principal step consists of replacing the original path of integration in the complex p-plane (the imaginary axis) by a path along which the exponential function in the integrands takes the form $\exp(-s\tau)$, where τ is a real variable of integration. In the present case, the modified path of integration is then defined through

$$px + \gamma_S(p)H = \tau, \tag{A.2}$$

in which τ is a real, positive parameter. From Eq. (A.2) it follows that the required modified path is given by $\{p = p_S(x,H,\tau)\} \cup \{p = p_S^*(x,H,\tau)\}$, where

$$p_S = \frac{x\tau}{r^2} + i\frac{H}{r^2}\left(\tau^2 - \frac{r^2}{c_S^2}\right)^{1/2} \quad \text{for} \quad r/c_S \leqslant \tau < \infty, \tag{A.3}$$

with

$$r = (x^2+H^2)^{1/2} \tag{A.4}$$

as the distance from the point with coordinates $\{0,0\}$ to the point with coordinates $\{x,H\}$ and where * denotes complex conjugate. Eq. (A.3) represents a hyperbolic arc in the upper half of the complex p-plane that intersects the real p-axis at the point $p = (x/r)c_S^{-1}$ at the parameter value $\tau = T_S$, with

$$T_S = r/c_S \tag{A.5}$$

as the SH-wave travel time over the distance r. The relevant point of intersection lies always in the interval $-c_S^{-1} \leqslant p \leqslant c_S^{-1}$ (Fig. 9).

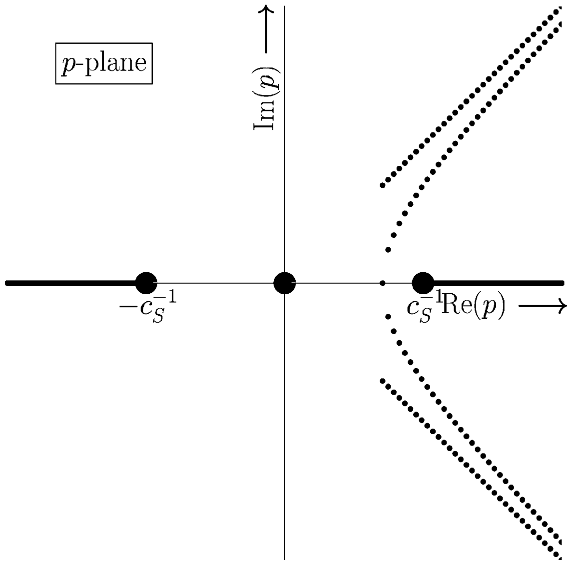

Fig. 9. Modified Cagniard path, with asymptote, in the complex slowness plane at equidistant values of τ.

The integral on the right-hand side of Eq. (A.1) is now replaced by the corresponding one along the modified Cagniard path $\{p = p_{\mathrm{S}}(x, H, \tau)\} \cup \{p = p_{\mathrm{S}}^*(x, H, \tau)\}$ and thereby retains its value in view of the regularity of the integrand in between this path and the original path of integration and the integrand's behavior at infinity. On account of this, Cauchy's theorem holds and Jordan's lemma applies to the connecting circular arcs at infinity. Along the path, the contributions from $p = p_{\mathrm{S}}(x, z, \tau)$ and $p = p_{\mathrm{S}}^*(x, z, \tau)$ are taken together under the application of Schwarz's reflection principle of complex function theory and τ is introduced as the variable of integration. The Jacobian of the change in variable of integration follows from Eq. (A.3) as

$$\frac{\partial p_{\mathrm{S}}}{\partial \tau} = \frac{\mathrm{i}H\tau}{r^2}\left(\tau^2 - \frac{r^2}{c_{\mathrm{S}}^2}\right)^{-1/2} + \frac{x}{r^2}, \tag{A.6}$$

which, with

$$\gamma_{\mathrm{S}}(p_{\mathrm{S}}) = \frac{H\tau}{r^2} - \mathrm{i}\frac{x}{r^2}\left(\tau^2 - \frac{r^2}{c_{\mathrm{S}}^2}\right)^{1/2}, \tag{A.7}$$

can be rewritten as

$$\frac{\partial p_{\mathrm{S}}}{\partial \tau} = \frac{\mathrm{i}\gamma_{\mathrm{S}}(p_{\mathrm{S}})}{(\tau^2 - r^2/c_{\mathrm{S}}^2)^{1/2}}. \tag{A.8}$$

Under these operations, Eq. (A.1) is replaced by

$$\hat{w}(x, z, s) = \int_{\tau=T_{\mathrm{S}}}^{\infty} \exp(-s\tau)\frac{\mathrm{Re}[A(p_{\mathrm{S}}, s)]}{2\pi(\tau^2 - T_{\mathrm{S}}^2)^{1/2}}\,\mathrm{d}\tau. \tag{A.9}$$

These results are used in the main text.

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